Turbulent Statistics from Time-Resolved PIV Measurements of a Jet Using Empirical Mode Decomposition

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Empirical mode decomposition is an adaptive signal processing method that when applied to a broadband signal, such as that generated by turbulence, acts as a set of band-pass filters. This process was applied to data from time-resolved, particle image velocimetry measurements of subsonic jets prior to computing the second-order, two-point, space-time correlations from which turbulent phase velocities and length and time scales could be determined. The application of this method to large sets of simultaneous time histories is new. In this initial study, the results are relevant to acoustic analogy source models for jet noise prediction. The high frequency portion of the results could provide the turbulent values for subgrid scale models for noise that is missed in large-eddy simulations. The results are also used to infer that the cross-correlations between different components of the decomposed signals at two points in space, neglected in this initial study, are important.

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18th AIAA/CEAS Aeroacoustics Conference
June 4–6, 2012

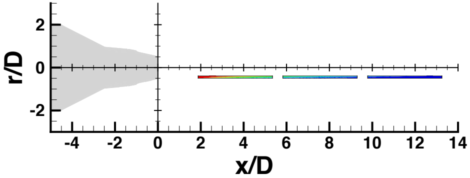
Supported by NASA Fundamental Aeronautics/Supersonics Project

Introduction

- Provide statistical properties of turbulence to aid in modeling jet noise sources
  - Acoustic analogy methods and subgrid models
  - Two-point, space-time correlations
    - numerically – direct numerical simulation
    - experimentally – time-resolved PIV
- Method – empirical mode decomposition
  - Automatic filtering for small scales – subgrid models
- Presentation:
  1. Time-resolved PIV measurements
  2. Empirical mode decomposition – new application
  3. Initial analysis of and results from correlation calculations
  4. Comments on method and correlations

Overview of Time-Resolved PIV

High sampling rate means small field of view



25 kHz
$\sim$1 sec, 24993 points
178.85 mm by 5.18 mm
70 by 5 points
$\Delta x/D = 0.0510$
$\Delta r/D = 0.0255$

Flow conditions for converging nozzle D = 50.8 mm

Case	T_t/T_∞	T_s/T_∞	P_t/P_∞	M_J	U_J/c_∞	$U_J(\mathrm{m/s})$	fU_J/D
SP3	**1.00**	**0.95**	**1.20**	**0.51**	**0.50**	**172.8**	**3.67**
SP7	**1.00**	**0.84**	**1.85**	**0.98**	**0.90**	**310.0**	**2.05**

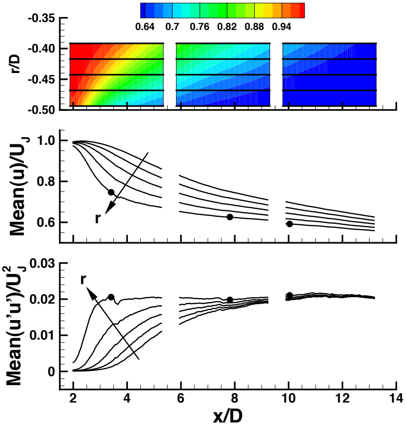
Mean Flow

SP7 jet with $M_J = 0.98$

Axial velocity contours

Axial velocity

Turbulence intensities



Reference points: $r/D = -0.49$

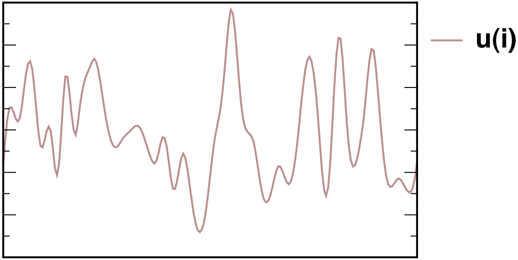
$x/D = 3.41, 7.82, 10.0$

Empirical Mode Decomposition (EMD)

- Adaptive signal processing method for a general, non-stationary and nonlinear signal
- Huang, et al.: Proc. Roy. Society London, 1998
- Separates signal into basis functions (a posteriori, based on the signal, not a priori, based on harmonics)
- Defined by an algorithm, limited mathematical foundation
- Applied in vibrations, geophysics, biology, cosmology, finance, others
- Application to large set of simultaneous time histories all having the same broadband nature and then computing correlations is new

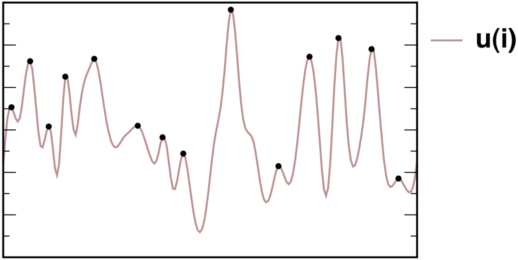
EMD Algorithm I

Start with the signal: the axial velocity fluctuation after subtracting the mean axial velocity



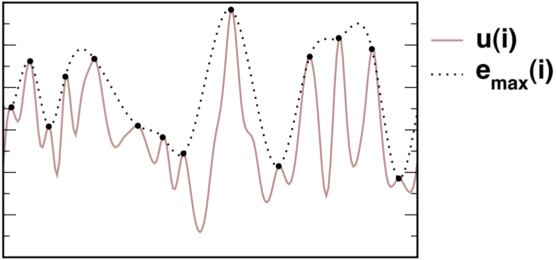
EMD Algorithm I

Identify all maxima of the signal



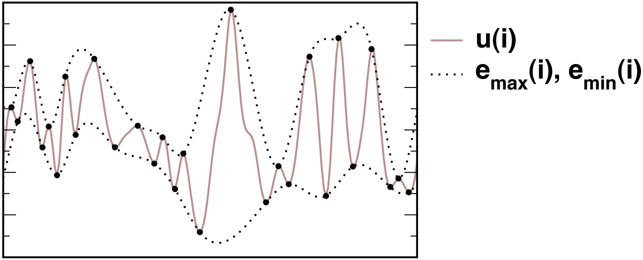
EMD Algorithm I

Interpolating function found using cubic spline gives maximum of envelope



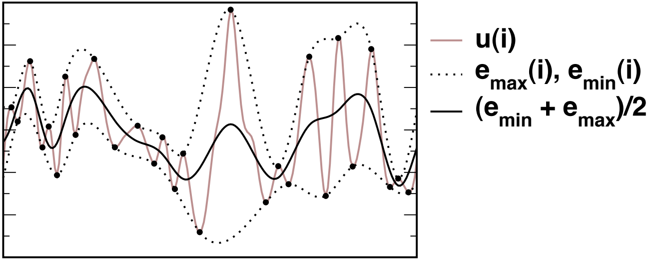
EMD Algorithm I

Repeat for the minimum envelope



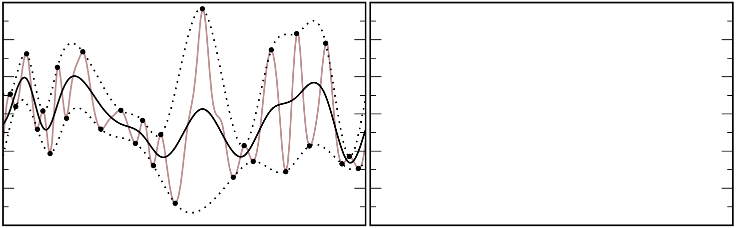
EMD Algorithm I

Compute average of the envelope



EMD Algorithm II

An interpretation of EMD is that it separates the local (high frequency) oscillations from the local (low frequency) trend



EMD Algorithm II

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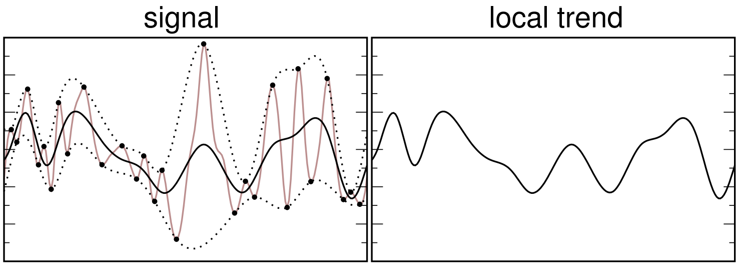
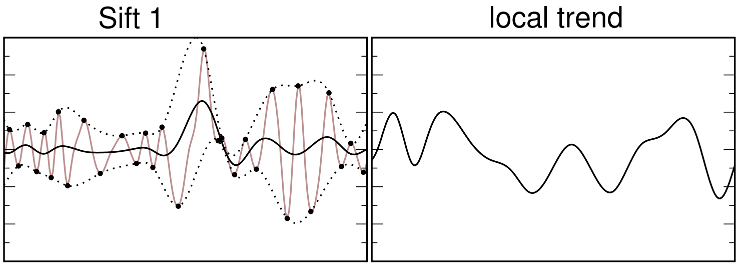


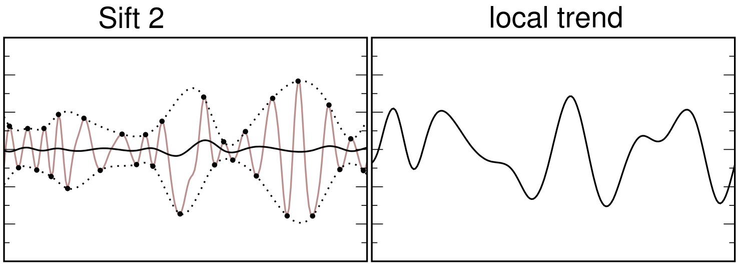
EMD Algorithm II

Subtract local trend from signal
Ideally should get zero mean oscillation



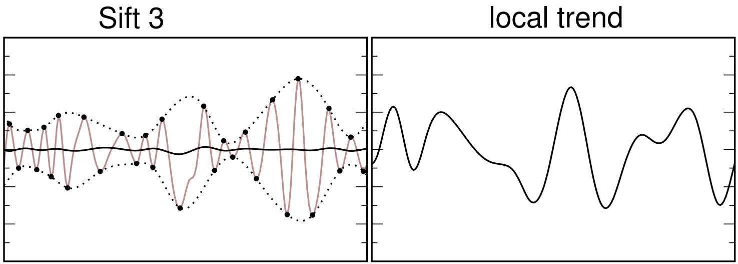
EMD Algorithm II

Subtract local trend from signal
Keep sifting



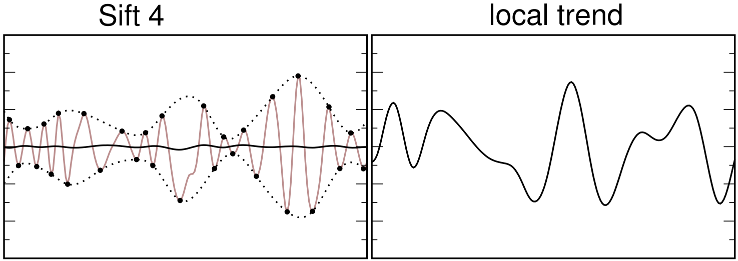
EMD Algorithm II

Subtract local trend from signal
Keep sifting



EMD Algorithm II

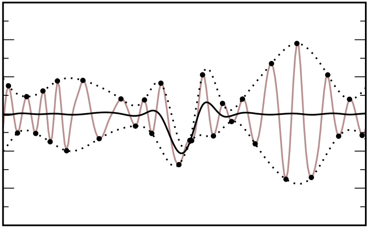
Subtract local trend from signal
Keep sifting



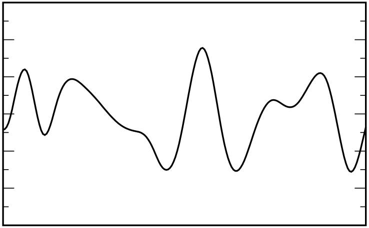
EMD Algorithm II

Subtract local trend from signal
Keep sifting

Sift 5

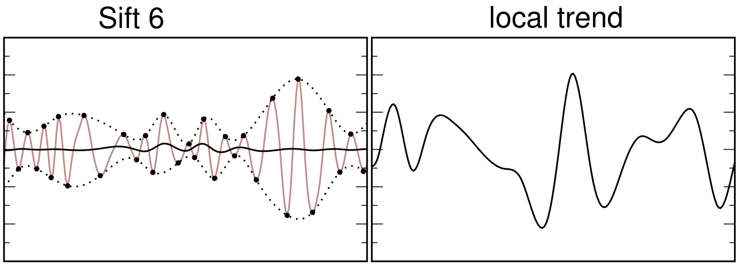


local trend



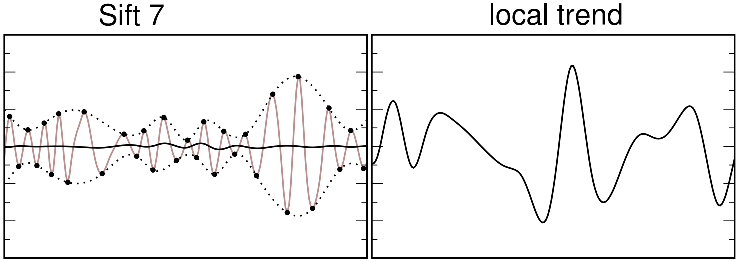
EMD Algorithm II

Subtract local trend from signal
Keep sifting



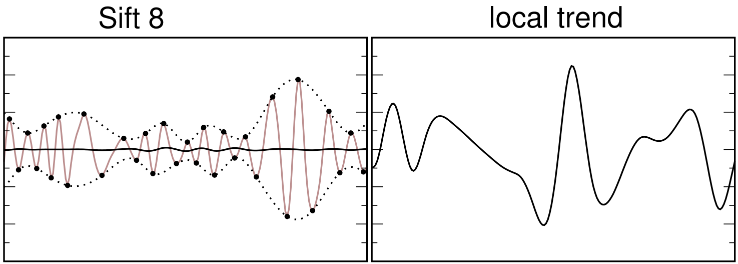
EMD Algorithm II

Subtract local trend from signal
Keep sifting



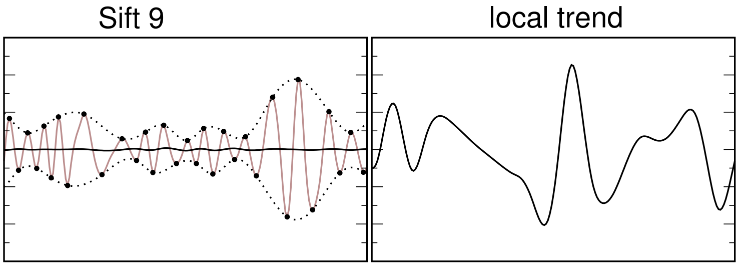
EMD Algorithm II

Subtract local trend from signal
Keep sifting



EMD Algorithm II

Subtract local trend from signal
Keep sifting

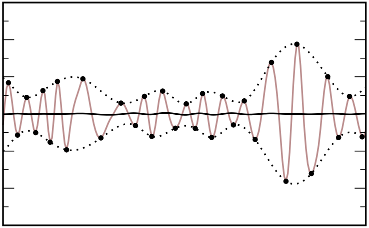


EMD Algorithm II

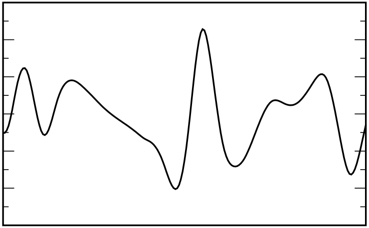
Subtract local trend from signal

Stop sifting

Sift 10



local trend



Intrinsic Mode Function (IMF)

– |(extrema) - (zeroes)| $\leq$ 1

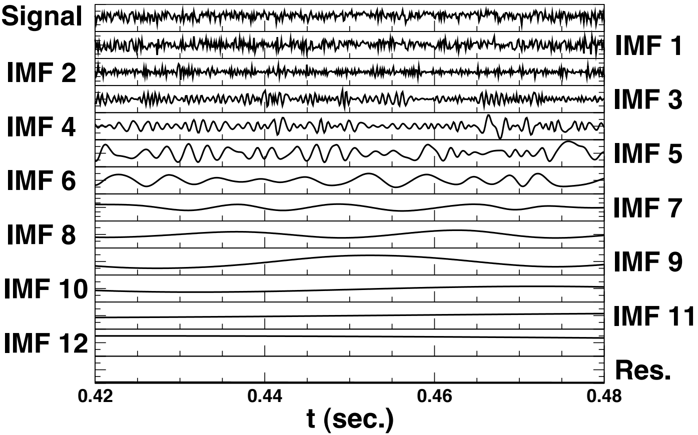
– Mean about zero

– Symmetric

Process with EMD to get next IMF

Example Intrinsic Mode Functions (IMF)

0.06 second time sample of a 1 second TR-PIV time history

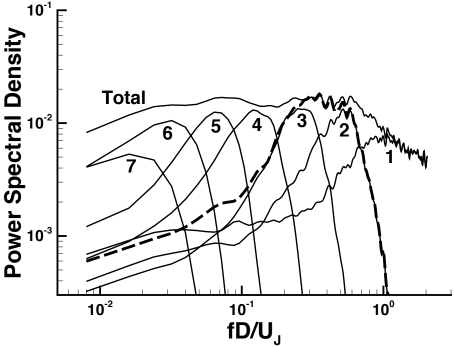


IMFs nearly orthogonal and uncorrelated: $u(t) = \sum_{n=1}^{N} C_n(t) + z_N(t)$

Broadband Signal Processed by EMD

EMD acts as a dyadic filter bank

- Set of overlapping band-pass filters
- half or double range of neighbor
- approx. constant shape on log scale
- mean frequencies $f_n^c \approx f_o 2^{-n}$ for n-th IMF



Dashed line:
Add IMFs 2 and 3

Ref. Point: x/D = 3.41 shear layer

Broadband Signal Processed by EMD

EMD acts as a dyadic filter bank

- Set of overlapping band-pass filters
- half or double range of neighbor
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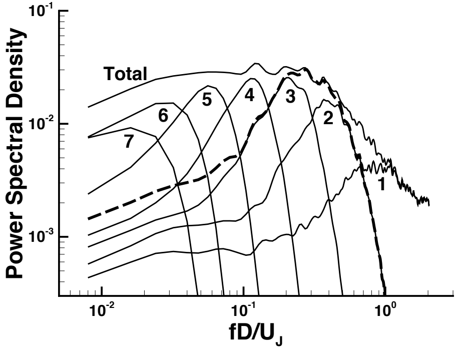
- mean frequencies $f_n^c \approx f_o 2^{-n}$ for n-th IMF



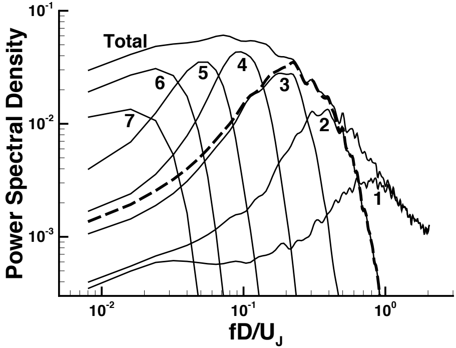
Dashed line:
Add IMFs 2 and 3

Ref. Point: x/D = 7.82 end of potential core

Broadband Signal Processed by EMD

EMD acts as a dyadic filter bank

- Set of overlapping band-pass filters
- half or double range of neighbor
- approx. constant shape on log scale
- mean frequencies $f_n^c \approx f_o 2^{-n}$ for n-th IMF

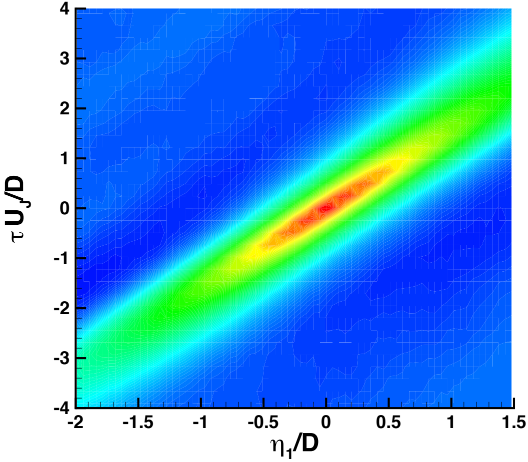


Dashed line:
Add IMFs 2 and 3

Ref. Point: x/D = 10.0 downstream

Correlation I

Total $r_{11}(\boldsymbol{x},\eta_1,\tau) = \dfrac{\overline{u_1'(\boldsymbol{x},t)\cdot u_1'(\boldsymbol{x}+\eta_1,t+\tau)}}{\sqrt{\overline{u_1'(\boldsymbol{x},t)^2}\cdot\overline{u_1'(\boldsymbol{x}+\eta_1,t+\tau)^2}}}$

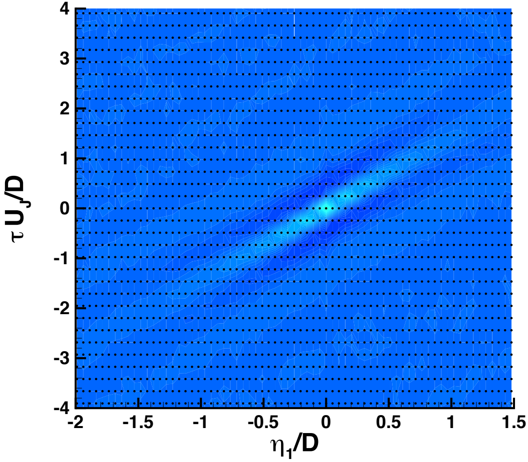


scale: -0.1 to 1

Ref. Pt.:
x/D = 7.82
r/D = -0.49

Correlation I

IMF 1 $\quad r_{11n}(\boldsymbol{x},\eta_1,\tau) = \dfrac{\overline{C_{1n}(\boldsymbol{x},t)\cdot C_{1n}(\boldsymbol{x}+\eta_1,t+\tau)}}{\sqrt{\overline{u_1'(\boldsymbol{x},t)^2}\cdot\overline{u_1'(\boldsymbol{x}+\eta_1,t+\tau)^2}}}, n = 1$



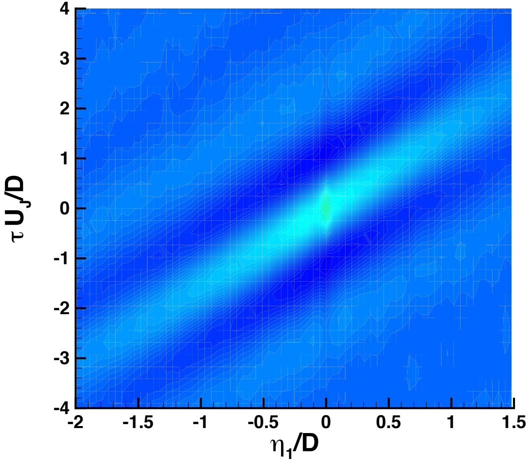
scale: -0.1 to 1

Ref. Pt.:
x/D = 7.82
r/D = -0.49

Grid too coarse?

Correlation I

IMF 2 $$r_{11n}(\boldsymbol{x},\eta_1,\tau)=\frac{\overline{C_{1n}(\boldsymbol{x},t)\cdot C_{1n}(\boldsymbol{x}+\eta_1,t+\tau)}}{\sqrt{\overline{u_1'(\boldsymbol{x},t)^2}\cdot\overline{u_1'(\boldsymbol{x}+\eta_1,t+\tau)^2}}},n=2$$

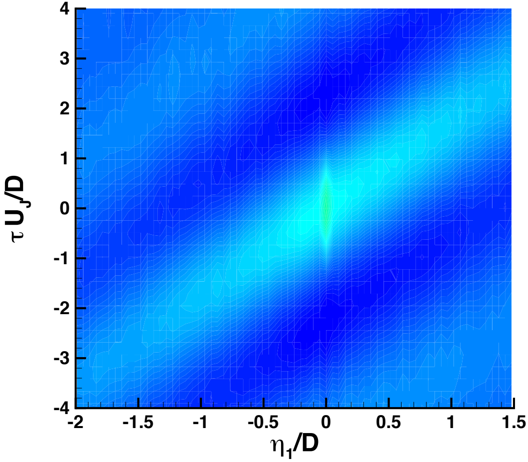


scale: -0.1 to 1

Ref. Pt.:
x/D = 7.82
r/D = -0.49

Correlation I

IMF 3 $\quad r_{11n}(\boldsymbol{x},\eta_1,\tau) = \dfrac{\overline{C_{1n}(\boldsymbol{x},t)\cdot C_{1n}(\boldsymbol{x}+\eta_1,t+\tau)}}{\sqrt{\overline{u_1'(\boldsymbol{x},t)^2}\cdot\overline{u_1'(\boldsymbol{x}+\eta_1,t+\tau)^2}}}, n=3$

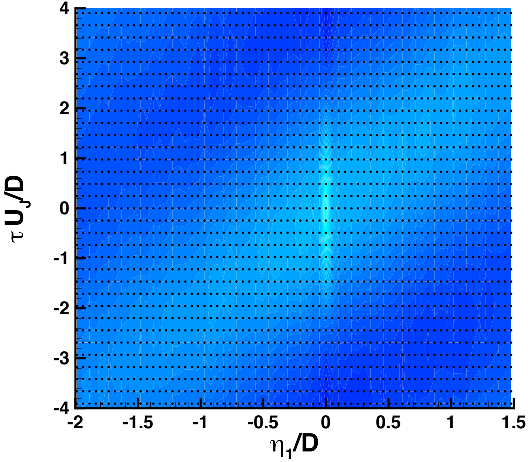


scale: -0.1 to 1

Ref. Pt.:
x/D = 7.82
r/D = -0.49

Correlation I

IMF 4 $\quad r_{11n}(\boldsymbol{x},\eta_1,\tau) = \dfrac{\overline{C_{1n}(\boldsymbol{x},t)\cdot C_{1n}(\boldsymbol{x}+\eta_1,t+\tau)}}{\sqrt{\overline{u_1'(\boldsymbol{x},t)^2}\cdot\overline{u_1'(\boldsymbol{x}+\eta_1,t+\tau)^2}}}, n=4$

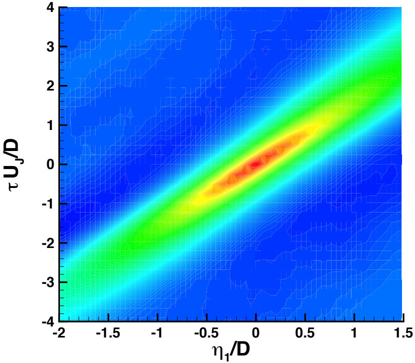


scale: -0.1 to 1

Ref. Pt.:
x/D = 7.82
r/D = -0.49

Issue within 1 grid spacing

Integral Length and Time Scales

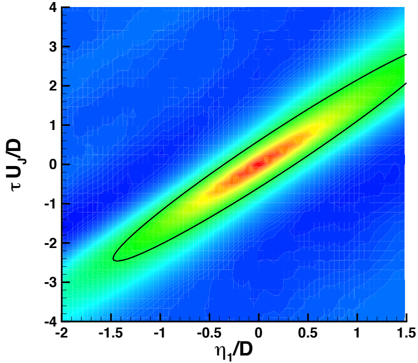


$$L_\eta = \int_0^{+\infty} r_{11}(\boldsymbol{x}, \eta_1, \tau = 0)\, \mathrm{d}\eta_1$$

$$\tau_\eta = \int_0^{+\infty} r_{11}(\boldsymbol{x}, \eta_1 = U_c\tau, \tau)\, \mathrm{d}\tau$$

Estimate the integral length and time scales by determining how far it takes the correlation to decay by 1/e from the peak value.

Integral Length and Time Scales



$$L_\eta = \int_0^{+\infty} r_{11}(\boldsymbol{x}, \eta_1, \tau = 0)\, \mathrm{d}\eta_1$$

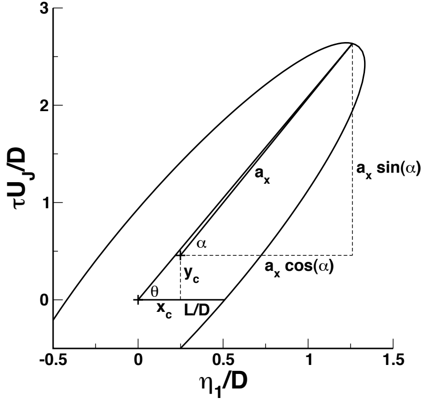
$$\tau_\eta = \int_0^{+\infty} r_{11}(\boldsymbol{x}, \eta_1 = U_c\tau, \tau)\, \mathrm{d}\tau$$

Estimate the integral length and time scales by determining how far it takes the correlation to decay by 1/e from the peak value. Contour resembles an ellipse.

Ellipse

Fit an ellipse to the contour where the level is 1/e times the peak value of the correlation. Find coefficients *a* to *f*.

$$a\left(\frac{\eta_1}{D}\right)^2 + b\frac{\eta_1}{D}\frac{\tau U_J}{D} + c\left(\frac{\tau U_J}{D}\right)^2 + d\frac{\eta_1}{D} + e\frac{\tau U_J}{D} + f = 0$$

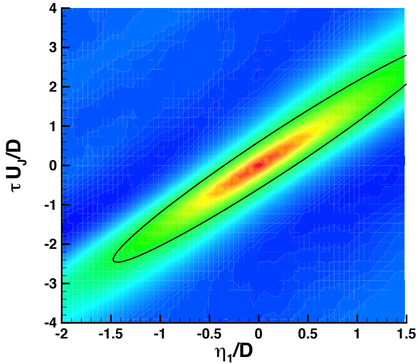


$$\frac{L}{D} = -\left(\frac{d}{2a}\right) + \sqrt{\left(\frac{d}{2a}\right)^2 - \frac{f}{a}}$$

$$\frac{\tau U_J}{D} = a_x \sin\alpha + y_c$$

$$\frac{U_c}{U_J} = \frac{1}{\tan\theta}$$

Correlation II



Estimate:

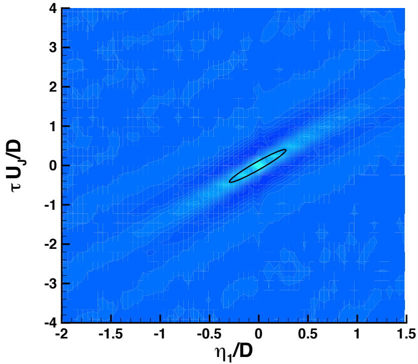
$$L_\eta = \int_0^{+\infty} r_{11}(\boldsymbol{x}, \eta_1, \tau = 0)\,\mathrm{d}\eta_1$$

$$\tau_\eta = \int_0^{+\infty} r_{11}(\boldsymbol{x}, \eta_1 = U_c\tau, \tau)\,\mathrm{d}\tau$$

using ellipse eq.

	Total
U_c/U_J	**0.596**
L_η/D	**0.366**
$\tau_\eta U_J/D$	**3.057**

Correlation II



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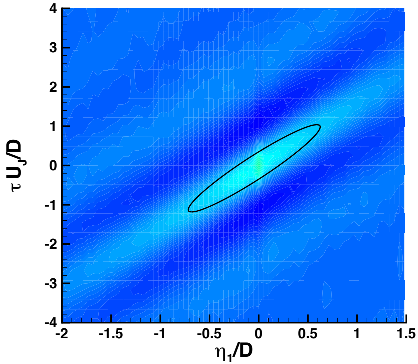
using ellipse eq.

	Total	**IMF 1**
U_c/U_J	**0.596**	**0.670**
L_η/D	**0.366**	**0.076**
$\tau_\eta U_J/D$	**3.057**	**0.403**

Correlation II



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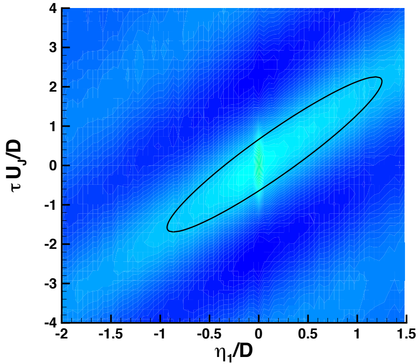
$$\tau_\eta = \int_0^{+\infty} r_{11}(\boldsymbol{x}, \eta_1 = U_c\tau, \tau)\, \mathrm{d}\tau$$

using ellipse eq.

	Total	**IMF 1**	**IMF 2**
U_c/U_J	**0.596**	**0.670**	**0.586**
L_η/D	**0.366**	**0.076**	**0.206**
$\tau_\eta U_J/D$	**3.057**	**0.403**	**1.036**

Correlation II



Estimate:

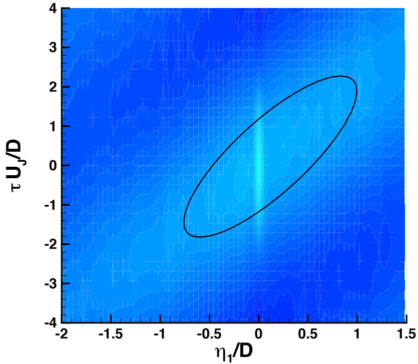
$$L_\eta = \int_0^{+\infty} r_{11}(\boldsymbol{x}, \eta_1, \tau = 0)\,\mathrm{d}\eta_1$$

$$\tau_\eta = \int_0^{+\infty} r_{11}(\boldsymbol{x}, \eta_1 = U_c\tau, \tau)\,\mathrm{d}\tau$$

using ellipse eq.

	Total	**IMF 1**	**IMF 2**	**IMF 3**
U_c/U_J	**0.596**	**0.670**	**0.586**	**0.540**
L_η/D	**0.366**	**0.076**	**0.206**	**0.375**
$\tau_\eta U_J/D$	**3.057**	**0.403**	**1.036**	**2.246**

Correlation II



Estimate:

$$L_\eta = \int_0^{+\infty} r_{11}(\boldsymbol{x}, \eta_1, \tau = 0)\, \mathrm{d}\eta_1$$

$$\tau_\eta = \int_0^{+\infty} r_{11}(\boldsymbol{x}, \eta_1 = U_c\tau, \tau)\, \mathrm{d}\tau$$

using ellipse eq.

	Total	**IMF 1**	**IMF 2**	**IMF 3**	**IMF 4**
U_c/U_J	**0.596**	**0.670**	**0.586**	**0.540**	**0.384**
L_η/D	**0.366**	**0.076**	**0.206**	**0.375**	**0.545**
$\tau_\eta U_J/D$	**3.057**	**0.403**	**1.036**	**2.246**	**2.266**

High frequency, small-scale for subgrid model

Frequency Dependent Values

Compute complex coherence γ_{ij} (Kerhervé et al. (2006))

Length scale $$\Lambda^1_{11}(\boldsymbol{x},\omega) = \int_0^{+\infty} \Re\left\{\gamma_{11}(\boldsymbol{x},\eta_1,\omega)\right\} \,\mathrm{d}\eta_1$$

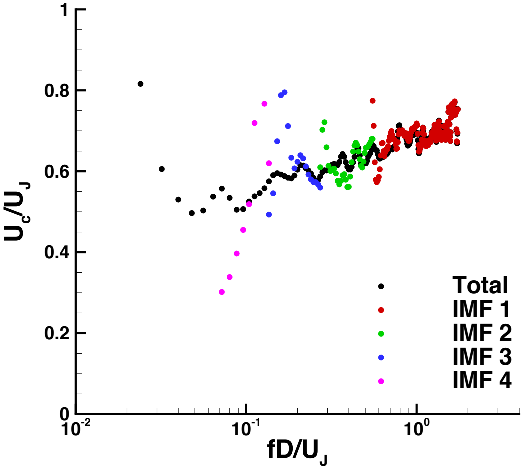
Time scale $$\tau^1_{11}(\boldsymbol{x},\omega) = \frac{1}{u_{c1}(\omega)} \int_0^{+\infty} \left|\gamma_{11}(\boldsymbol{x},\eta_1,\omega)\right| \,\mathrm{d}\eta_1$$

Phase velocity $$u_{c1}(\omega) = \omega / \left|\partial\phi(\eta_1,\omega)/\partial\eta_1\right|$$

- Scales difficult to compute without modeling, ex. $\Re\left\{\gamma_{11}\right\}$ oscillates, integration difficult
- Estimate length scale from $|\gamma_{11}|$ decays by 1/e
- Estimate time scale from $|\gamma_{11}|/u_{c1}$ decays by 1/e

Frequency Dependent Phase Velocity

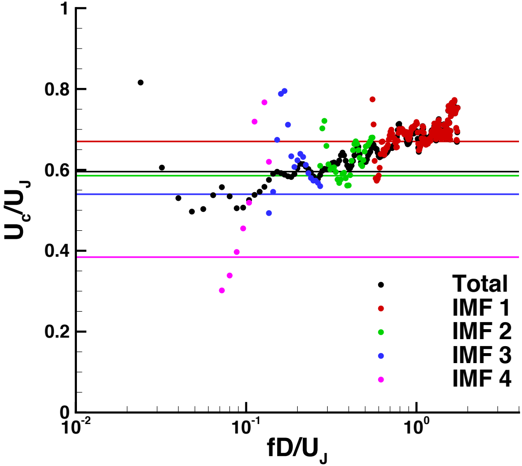
Issues: Higher IMFs (lower frequency) lack resolution, low number of averages



Ref. Pt.:
x/D = 7.82
r/D = -0.49
end of potential core

Frequency Dependent Phase Velocity

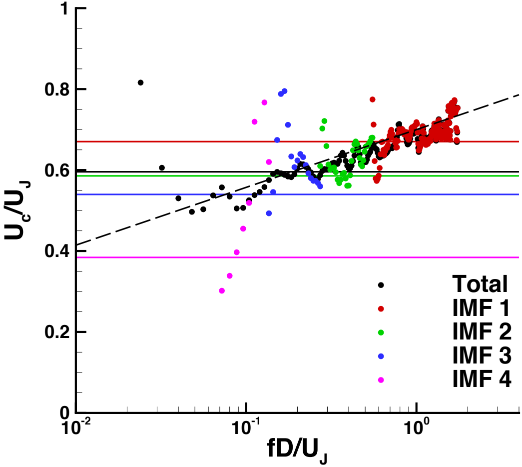
Compared to ellipse eq. based phase velocity



Ref. Pt.:
x/D = 7.82
r/D = -0.49
end of potential core

Frequency Dependent Phase Velocity

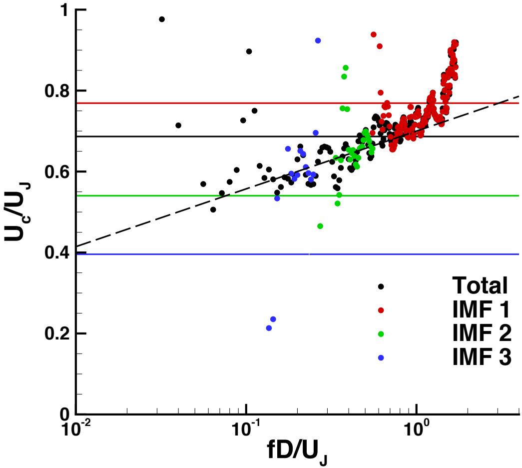
Compared to ellipse eq. based phase velocity
Compared to Morris & Zaman (2010) fit, M_J = 0.25 jet



Ref. Pt.:
x/D = 7.82
r/D = -0.49
end of potential core

Frequency Dependent Phase Velocity

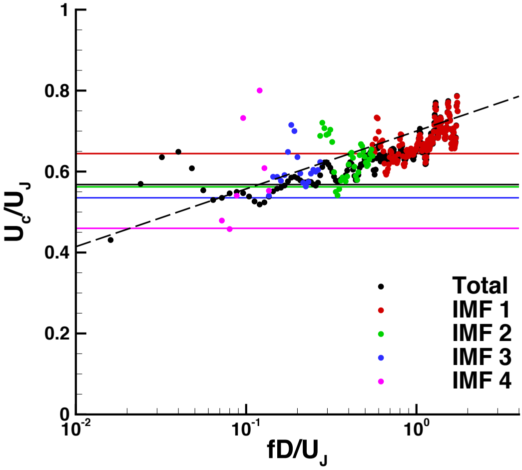
Compared to ellipse eq. based phase velocity
Compared to Morris & Zaman (2010) fit, M_J = 0.25 jet



Ref. Pt.:
x/D = 3.41
r/D = -0.49
shear layer

Frequency Dependent Phase Velocity

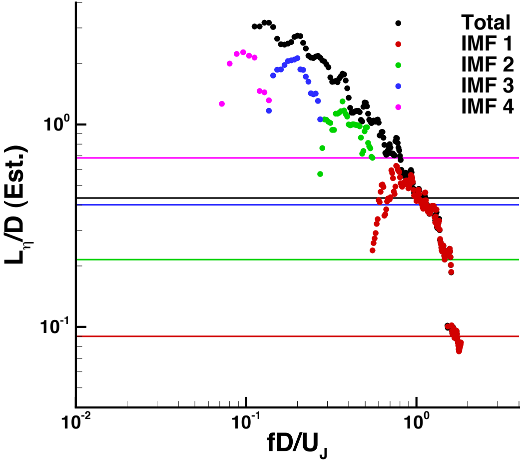
Compared to ellipse eq. based phase velocity
Compared to Morris & Zaman (2010) fit, M_J = 0.25 jet



Ref. Pt.:
x/D = 10.0
r/D = -0.49
downstream

Frequency Dependent Length Scale

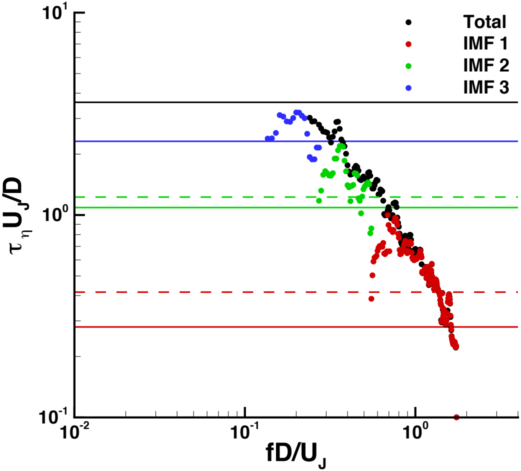
- Compared to ellipse eq. based length scale
- Using $|\gamma_{11}|$ instead of $\Re\{\gamma_{11}\}$ over-estimates the length scale



Ref. Pt.:
x/D = 10.0
r/D = -0.49
downstream

Frequency Dependent Time Scale

- Compared to ellipse eq. based time scale (solid)
- Direct interpolated value from correlation function (dashed)

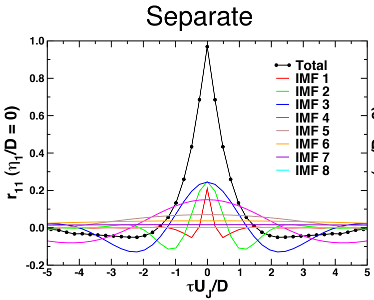


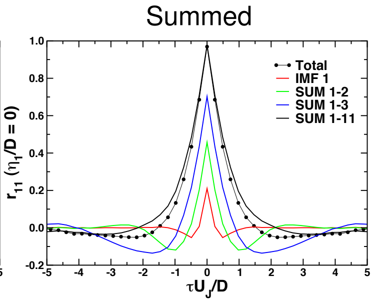
Ref. Pt.:
x/D = 10.0
r/D = -0.49
downstream

IMF Orthogonality and Correlation

Accepted that IMFs nearly orthogonal and uncorrelated at a point

$$r_{11}(\boldsymbol{x},\eta_1,\tau) = \frac{\overline{u_1'(\boldsymbol{x},t)\cdot u_1'(\boldsymbol{x}+\eta_1,t+\tau)}}{\sqrt{\overline{u_1'(\boldsymbol{x},t)^2}\cdot\overline{u_1'(\boldsymbol{x}+\eta_1,t+\tau)^2}}} = \sum_{n=1}^{N}\frac{\overline{C_{1n}(\boldsymbol{x},t)\cdot C_{1n}(\boldsymbol{x}+\eta_1,t+\tau)}}{\sqrt{\overline{u_1'(\boldsymbol{x},t)^2}\cdot\overline{u_1'(\boldsymbol{x}+\eta_1,t+\tau)^2}}}$$



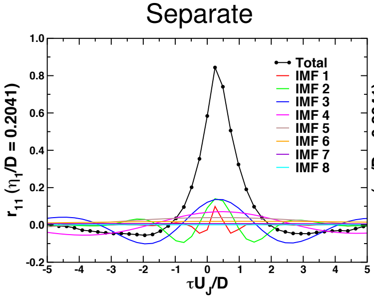


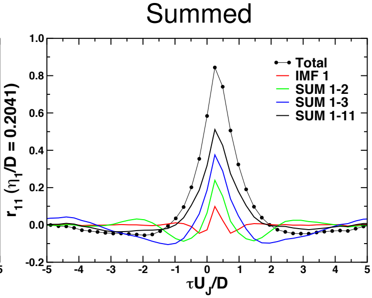
IMF Orthogonality and Correlation

What about the correlation between two points?

$$r_{11}(\boldsymbol{x},\eta_1,\tau) = \frac{\overline{u_1'(\boldsymbol{x},t)\cdot u_1'(\boldsymbol{x}+\eta_1,t+\tau)}}{\sqrt{\overline{u_1'(\boldsymbol{x},t)^2}\cdot\overline{u_1'(\boldsymbol{x}+\eta_1,t+\tau)^2}}} = \sum_{n=1}^{N}\frac{\overline{C_{1n}(\boldsymbol{x},t)\cdot C_{1n}(\boldsymbol{x}+\eta_1,t+\tau)}}{\sqrt{\overline{u_1'(\boldsymbol{x},t)^2}\cdot\overline{u_1'(\boldsymbol{x}+\eta_1,t+\tau)^2}}}$$

$+$ cross-correlation terms





Concluding Remarks

- New application of empirical mode decomposition
  - equivalent to passing array of time histories through a bank of band-pass filters – broadband signals only
  - highest frequency range turbulent statistics may be applicable to subgrid modeling
- Issues:
  - higher sampling rates: higher Strouhal numbers, finer time resolution at high frequencies
  - longer time histories: greater frequency resolution, increased number of averages
  - finer spatial resolution: capture high frequency changes in a short distance
- Further analysis:
  - IMF cross-correlation terms
  - Other approaches to using EMD results

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